

University of Southern Queensland



Steady and Unsteady Free Surface Flow Past a Two-Dimensional Stern

A thesis submitted in fulfilment of

Doctor of Philosophy

Osama Nasser Ogilat

B.Sc., M.Sci.

2013

Abstract

The research examines an influence of a platform shape on free surface waves generated behind a semi-infinite two-dimensional platform moving with a constant speed on a water surface of a finite depth h . The fluid is assumed to be inviscid, incompressible and irrotational; the surface tension effect is neglected. The aim of the research is to find analytically and numerically such platform shape which minimizes generated waves and reduces wave drag exerting on a moving platform when the Froude number is relatively small, $F < 1$. It is shown that for certain platform shapes, generated waves can be minimised or even eliminated, at least, within the framework of a linearized theory.

Linearized hydrodynamic equations for a fluid of finite depth are solved analytically by means of the Fourier transform and Wiener–Hopf technique, as well as numerically with the help of boundary integral technique. A weakly nonlinear solution is also obtained for shallow-water approximation within the framework of the forced Korteweg–de Vries (KdV) equation.

The problem is investigated for steady motion of a platform having a different stern shape. Then the analysis is performed for unsteady motion of a platform having a flat shape. The linearized problem for a water of finite depth is solved by means of the Laplace transform and Wiener–Hopf technique. The linear problem is formulated by assuming that at the initial instant of time the free surface is slightly perturbed due to the platform submerging onto the depth $d \ll h$ beneath the free surface. It is shown that the unsteady solution approaches the steady state solution as $t \rightarrow \infty$. The dependence of maximum wave perturbation on the fluid depth is found numerically.

In the last Chapter 6 the analysis is extended to steady motion of a flat platform at the interface between two fluids of different density. It is assumed that the lower layer has a finite depth h , whereas the upper layer is infinite. Results obtained for internal waves on the sharp density interface depend on the density ratio $a = \rho_1/\rho_2$ and in the limit $a \rightarrow 0$ they coincide with the results obtained for surface waves.

The results of this research can help in understanding of the physics of wave generation past a bluff body (e.g., wide blunt ships) and shed some light on solving an engineering problem of ship building of an optimal shape.

Statement of Original Authorship

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher educational institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

Candidate: Osama Nasser Ogilat

Signed: _____

Date: _____

ENDORESMENT

Principal supervisor: Associate Professor Yury Stepanyants

Signed: _____

Date: _____

Acknowledgements

My first and foremost thanks to ALLAH for the opportunities that He has given to me throughout my life, especially those that have brought me to the position of finishing this thesis.

I would like to express my thankfulness and gratitude to my principle supervisor Professor Yury Stepanyants for his invaluable assistance, support, patience and guidance during the period of my research. My special thanks are addressed to my associate supervisor Dr Dmitry Strunin for his advice, support and constructive feedback. Deepest gratitude is also due to Professor Ian Turner, Dr Scott McCue and Professor John Belward without their knowledge and assistance this study would not have been successful. Warmest thanks to Dr Ben Binder for his contribution to our joint paper published in Physics of Fluids. I would like to thank all postgraduate students and staff of the Department of Mathematics and Computing of USQ for providing a very good scientific environment for mathematical research.

I am indebted to the most important person in my live, my mother “Gazeah”, for her incredible support, love and patience through my many years in education. My special thanks to my wonderful brothers, Khaled, Amjad, Ahmad and Moa’ath and sisters, Manal, Tharwah, Khawlah, Hend, Wisam and Ebtsam who always wished my success. I am also thankful to my sisters-in-law and brothers-in-law who always supported me. I should not forget my relatives (uncles and cousins), as well as friends.

This thesis is dedicated to my mother; and to the soul of my father (ALLAH bless upon him), that I wish he is alive to see what I have achieved and to share my happiness for completing this Theses, who encouraged and directed me for education and provided me with safe financial living before and after he passed away.

Keywords

Boundary integral technique, Wiener–Hopf technique, weakly nonlinear theory, free surface flow, steady flow, unsteady flow, Fourier transform, Laplace transform, conformal mapping, Korteweg–de Vries (KdV) equation.

List of Publications

O. Ogilat, S. W. McCue, I. W. Turner, J. A. Belward, and B. J. Binder. Minimising wave drag for free surface flow past a two-dimensional stern. *Physics of Fluids*, 2011, v. 23, p. 072101.

O. Ogilat, Y. Stepanyants. Transient free surface flow past a two-dimensional flat stern. (*Accepted in Physics of Fluids journal*).

O. Ogilat, Y. Stepanyants. Minimising internal wave generation past steadily moving flat platform in two-layer fluid of finite depth. (*To be submitted to the journal Physics of Fluids*).

Contents

1	Introduction and Literature Review	15
1.1	Two-dimensional free surface flow past a semi-infinite stern	16
1.1.1	Steady flows in fluid of infinite depth	17
1.1.2	Steady flows in fluid of finite depth	18
1.1.3	Unsteady two-dimensional flow	19
1.1.4	Steady two-dimensional flow for the two-layer model	19
1.2	Techniques	20
1.2.1	Wiener–Hopf technique	20
1.2.2	Boundary-integral-equation technique	21
1.3	Research objectives	21
1.4	Content of this research	23
2	Exact Solution to the Linearised Steady Problem	25
2.1	Introduction	25
2.2	Mathematical Formulation	26
2.3	Wiener–Hopf technique	30
2.3.1	Application Of The Fourier Transform	30
2.3.2	The Wiener–Hopf equation	33
2.4	Case Studies	40
2.5	Numerical Considerations	45
2.6	Results	47
3	Numerical Solution to the Fully Nonlinear Steady Problem	55
3.1	Introduction	55

3.2	Problem Formulation	56
3.2.1	Conformal Mapping	57
3.2.2	Boundary Integral equation	61
3.2.3	Numerical Procedure	64
3.2.4	Free Surface Profiles	66
3.2.5	Results	67
4	Weakly Non-linear Theory	74
4.1	Introduction	74
4.2	Weakly Nonlinear Theory	75
4.3	Results for the Flat Plate Case	78
4.4	Geometry Treated By Binder (2010)	83
4.5	Results for a Family Of Curved Plates	94
4.5.1	Upward Pointing Sterns	94
4.5.2	Downward Pointing Sterns	96
5	Linearised Unsteady Flow Problem	107
5.1	Introduction	107
5.1.1	Mathematical Formulation	108
5.1.2	Linearised Stern Flow Problem	112
5.2	Wiener–Hopf technique	113
5.2.1	Application of the Laplace and Fourier Transforms	113
5.2.2	The Wiener–Hopf equation	116
5.2.3	The Location of the Free Surface $\eta_1(x, t)$	117
5.2.4	Numerical Approximation of the Contour Integral	118
5.2.5	Factorisation of $G_1(k, s)$	122
5.3	Results and Discussions	130
5.4	Conclusion	131
6	Stationary Internal Waves Past a 2D Stern in Two-Layer a Fluid	137
6.1	Introduction	137
6.2	Mathematical formulation	137
6.2.1	Linearised two-layer model	141

6.2.2	Application of Fourier Transforms	142
6.3	The Wiener–Hopf equation	143
6.3.1	Factorisation of $G_2(k)$	145
6.4	Calculation of the interface shape $\eta_1(x, t)$	147
6.5	Conclusion	148
7	Conclusion and Future Work	151
7.1	Research Outcomes	151
7.2	Future Directions	153
A	Relation Between P and ϵ	154
B	Consideration of the Infinite Product $T(k)$	155
C	Evaluation $I(x, z/t)$ and The Roots Of $f_1(k, z/t)$ For Different Values Of z	160

List of Figures

2.1	A schematic of the free surface flow past a semi-infinite curved plate in a finite depth fluid.	26
2.2	Figure (a) shows the strip $0 < \text{Im}(k) < \tau_+$. Figure (b) shows the path of integration in the lower half k -plane.	31
2.3	Free surface profiles drawn for flat plate shapes given by (2.62) for different scales, with $F = 0.5$ (red), and $F = 0.7$ (black) by using Method 1 in Section 2.3.2. These figures are presented exactly by McCue and Stump [85] on the same scale.	49
2.4	The dependence of the wavelength λ on the Froude number F . The red dashed is computed using dispersion relation and the black solid line is the asymptotic line.	50
2.5	Free surface profiles drawn for the plate shape in Case 2 given by (2.64), with $F = 0.5$, $a = 1$ and $b = 2$ (Figure (a)) and $F = 0.7$, $a = 1$ and $b = 2$ (Figure (b)), by using Method 1 (circles) and Method 2 (solid-line) as given by (2.68) and (2.72), respectively, in Section 2.3.2.	51

- 2.6 Figures (a) and (b) show the dependence of the amplitude A on the parameter a with $b = 1$ (black), 2(red), 3(green), and the dependence of the amplitude A on b for $a = 1$ (black), 2(red), 3(green), for $F = 0.5$, respectively, for the plate shape given by (2.64). Figure (c) shows the free surface for the local minimum of the parameter a , $(a = 1, b = 1, A = 0.298)$ (black), $(a = 1.9, b = 2, A = 0.549)$ (red), $(a = 2.5, b = 3, A = 0.73)$ (green). Figure (d) shows the free surface for the local minimum of the b , $(a = 1, b = .9, A = 0.289)$ (black), $(a = 2, b = 1.8, A = 0.529)$ (red), $(a = 3, b = 2.6, A = 0.749)$ (green). 52
- 2.7 Free surface profile drawn for the plate shape (2.64) with $F = 0.5$ and $a = 0$ (solid black), 0.5(green dashed), 1(red dot-dashed), 1.5((blue dot)) and $b = 1$ 53
- 2.8 In figure (a), Free surface profiles are drawn for the plate shape in Case 3 given by (2.77), with $F = 0.5$, $a = 1$, $b = 2$, $L = 3$, using Method 1 in Section 2.3.2. In figures (b), (c) and (d) the dependence of the wave amplitude A on the parameters a , b and L respectively, with $F = 0.5$, $b = 1$ (black), 2(red), 3(blue) and $L = 1$ in figure (a), $a = 1$ (black), 2(red), 3(green) and $L = 1$ in figure (b) and $a = -0.5$ (red), 0(blue), 0.5(green), 1(black) and $b = 1$ in figure (c). 54
- 3.1 Sketch of the flow and the position of the coordinates 56
- 3.2 This figure shows the flow in the complex f -plane shown in figure 58
- 3.3 The flow in the complex ζ -plane shown in figure a 60
- 3.4 This figure shows the complex ζ -plane on the contour γ 61
- 3.5 This figure show the relation between the iteration and the error. 69
- 3.6 The free surface profile for $F = 0.5$, $P = 0.01$, $b = 1$ 70
- 3.7 The free surface profile for $F = 0.9$, $P = 0.01$, $b = 1$ 70
- 3.8 Figure (a) shows the numerical solution for $F = 0.8$, $P = 0.01$, $b = 1$, $a = 0, \dots, 0.2$ 71
- 3.9 Figure (a) shows the numerical solution for $F = 0.7$, $P = 0.01$, $b = 1$, $a = 0, \dots, 0.2$ 71

- 3.10 Figure (a) shows the numerical solution for $F = 0.6$, $P = 0.01$,
 $b = 1$, $a = 0, \dots, 0.2$ 72
- 3.11 Figure (a) shows the numerical solution for $F = 0.4$, $P = 0.01$,
 $b = 1$, $a = 0, \dots, 0.14$ 72
- 3.12 This figure shows the relation between the amplitude A and the
parameter a 73
- 3.13 This figure shows the relation between the amplitude A and the
Froude number F 73
- 4.1 Figures of the free surface, comparing the numerical (solid-lines),
analytical (dashed-lines), and weakly nonlinear (dot dashed-lines)
solutions for the case of the flat plate, with Froude number $F = 0.9$,
and $\epsilon = P/(1 - F^2)$ at (a) $P = 0.1$, at (b) $P = 0.01$, (c) $P = 0.001$,
and (d) $P = 0.0001$ 80
- 4.2 Figures of the free surface, comparing the numerical (solid-lines),
analytical (dashed-lines), and weakly nonlinear (dot dashed-lines)
solutions for the case of the flat plate, with Froude number $F = 0.5$,
and $\epsilon = P/(1 - F^2)$ at (a) $P = 0.1$, at (b) $P = 0.01$, (c) $P = 0.001$,
and (d) $P = 0.0001$. Note that, in (a), the numerical solution does
not converge, and thus is not shown. 81
- 4.3 Figures (a) and (b) show the comparison between the numerical so-
lution (solid-line), analytical solution (dashed-line), for the highest
value of P that the numerical solution allows. The Froude number
 $F = 0.5$ (figure (a)) and $F = 0.9$ (figure (b)), at $P = 0.05, 0.3$
respectively. 82
- 4.4 Figures (a) and (b) show the relation between the amplitude A and
the Froude number F for the case of the flat plate at $P = 0.01$,
for the analytical (dashed-line), numerical (red dots), and weakly
nonlinear solutions (blue dot), for different scale. 83

- 4.5 Figures (a) and (b) show the comparison between the numerical solution (dashed-line), analytical solutions in the solid-line for the case of the flat plate. The Froude number $\tilde{F} = 0.5$ and $\epsilon = 0.001, 0.058$ respectively, where $F = (1 - \epsilon)\tilde{F}$. These figures are as presented by McCue and Forbes in 2002. In figure (c) and (d) shows the comparison between analytical and numerical solutions for $F = 0.5$ and $P = 0.001, 0.058$ respectively. 84
- 4.6 The nonlinear profile for the case of the flat plate is shown in figure (a) and the phase trajectories in figure (b), at the Froude number $F = 0.9$ and $P = 0.01$ 85
- 4.7 The weakly nonlinear profile for the case of the flat plate are shown in figure (a) and the phase trajectories in figure (b), at the Froude number $F = 0.9$ and $P = 0.01$ 85
- 4.8 The dependence of the wave amplitude A on the parameter α for $F = 0.9$, $P = 0.01$. The red solid curve, black dashed curve and blue dot-dashed curve correspond to the fully nonlinear, linear and weakly nonlinear solution, respectively. In (a) the scale is such that the nonlinear amplitude appears to vanish at a value of α , but the scale in (b) suggests there is in fact a local minimum for which A is finite but small. 88
- 4.9 The nonlinear free surface profiles drawn for different scales, with $F = 0.9$, $P = 0.01$ at $\alpha = 0.019$ (red), 0.0655 (red), 0.0427 (blue) respectively. 89
- 4.10 The weakly nonlinear profiles are shown in figure (a), (c) and (e), and phase trajectories are shown in figure (b), (d) and (f), with the Froude number $F = 0.9$ and $P = 0.01$ at $\alpha = 0.057, 0.019$, and 0.038 , respectively. These figures are the same as figures drawn by Binder ([11], 2010). 90
- 4.11 This figure shows the relation between the amplitude A and the parameter α for Froude number $F = 0.5$ and $P = 0.01$. The red solid curve and black dashed curve correspond to the fully nonlinear and linear solution, respectively. 91

- 4.12 Figures (a) and (b) show the free surface profile for the numerical solution at the Froude number $F = 0.5$, $P = 0.01$ and $\alpha = 0.0134$ (local minimum), for a different scale with amplitude $A = 1.9220 \times 10^{-4}$ 91
- 4.13 Figures (a) and (b) show the free surface profile for the analytical solution at the Froude number $F = 0.5$, $P = 0.01$ and $\alpha = 0.0136$ (local minimum), for a different scale with amplitude $A = 2.6417 \times 10^{-4}$ 92
- 4.14 The dependence of the wave amplitude A on the parameter α for $F = 0.5$, $P = 0.04$. The red solid curve, black dashed curve and blue dot-dashed curved correspond to the fully nonlinear, linear and weakly nonlinear solutions, respectively. 92
- 4.15 The relation between the amplitude A and the Froude number F at $\alpha = 0.0427$, $P = 0.01$ are shown in this figure. The red solid curve, black solid curve and blue dot-dashed curved correspond to the fully nonlinear, linear and weakly nonlinear solution, respectively. 93
- 4.16 Free surface profiles drawn for $F = 0.5$, $P = 0.01$, $b = 1$ and $L = 1$.
(a) Linear solutions with $a = 0$ (solid black), 1.5 (green dashed), 3 (red dot-dashed) and 4.5 (blue dot). (b) Nonlinear solutions for $a = 0, \dots, 3.9$ 96
- 4.17 The dependence of the wave amplitude A on the parameter a for $F = 0.5$ and $P = 0.01$. The red solid curve, black solid curve and blue dot-dashed curved correspond to the fully nonlinear, linear and weakly nonlinear solutions, respectively. 97
- 4.18 This figure shows the comparison between the analytic, numerical and the weakly nonlinear solution. The free surface profile are shown in figures (a) and (c) and phase trajectories are shown in figure (b) and (d) with the Froude number $F = 0.5$ and $P = 0.01$ at $a = 1, 3.15$ respectively. 98
- 4.19 The dependence of the wave amplitude A on the Froude number F for $a = 3.15$, $b = 1$, $L = 1$ and $P = 0.01$ 99

- 4.20 The dependence of the wave amplitude A on the parameter a for $F = 0.9$ and $P = 0.01$. The red solid curve, black solid curve and blue dot-dashed curve correspond to the fully nonlinear, linear and weakly nonlinear solutions, respectively. 99
- 4.21 This figure shows the comparison between the analytic, numerical and the weakly nonlinear solution. The free surface profile are shown in figures (a), (c) and (e) and phase trajectories are shown in figure (b), (d) and (f), with the Froude number $F = 0.9$ and $P = 0.01$ at $a = 0.7, 0.45$, and 0.4 respectively. 100
- 4.22 The dependence of the wave amplitude A on the parameter β for the nonlinear solution for $F = 0.9$, $P = 0.01$, for the values $\alpha = 0.03$, 0.04 , 0.0427 and 0.05 101
- 4.23 The dependence of the wave amplitude A on the parameter β for $F = 0.5$, $P = 0.04$ and $\alpha = 0.0527$. The red solid curve and black dashed curve correspond to the fully nonlinear and linear solution, respectively. 102
- 4.24 Nonlinear free surface profiles drawn for $F = 0.5$, $P = 0.04$ and $\alpha = 0.0527$ with $\beta = -0.005$, 0.0004 and 0.005 with amplitude $A = 0.0029$, 0.00005 and 0.0029 , respectively. 102
- 4.25 The dependence of the wave amplitude A on the parameter β for $F = 0.5$, $P = 0.01$ and different values of α , $\alpha = -0.008$, -0.002 , 0 , 0.002 , 0.00135 . (a) nonlinear solution, (b) linear solution. 103
- 4.26 Figure (a) and (b), show the free surface profile for different scale, for the analytical and numerical solution. The Froude number $F = 0.5$, $P = 0.01$, $\alpha = 0.00135$ with the local minimum $\beta = 0.004$ 103
- 5.1 Sketch of free surface flow past a semi-infinite flat plate in a fluid of finite depth. In figure (a), the plate is located at the level of the undisturbed free surface, whereas in figure (b) the plate is suddenly submerged into the fluid for $\tilde{t} > 0^+$ 108

- 5.2 A schematic of the free surface flow past a flat plate with velocity potential upstream given by $\Phi = V\tilde{x}$ and velocity potential downstream given by $\Phi = (\frac{Vh}{h+d})\tilde{x}$ 110
- 5.3 A schematic for the non-dimensional unsteady problem for $t > 0^+$, where $\epsilon = d/h$ is the non-dimensional distance between the height of the plate and the undisturbed free surface. 112
- 5.4 The contour of integration consisting of the path Γ' and path c of infinitely large radius R . Red dots show the positions of poles in equation (5.61). 120
- 5.5 Figure (a) shows plot of the function $G_1(k)$ in equation (5.73) at $F = 0.5$. Figure (b) shows plot of the function $G_1(k)$ in equation (5.79) at $F = 0.5$. Figure (c) shows plot of the function $G_1(k)$ in equation (5.73) and (5.79). 125
- 5.6 Zero isolines of functions $\text{Ref}_1(k, z/t)$ (red lines) and $\text{Im}f_1(k, z/t)$ (blue lines). Dots indicate intersection points of isolines. The plot was generated for $F = 0.5$, $t = 1$ and $z_1 = 4.0277 + 1.1939i$ 126
- 5.7 Free surface profile in dimensionless variables relative to unperturbed level $y = 1$ for $t = 1000$ as obtained from equation (5.71) with different numbers of poles in Table 5.1. Dashed line pertains to $N = 8$, solid line – $N = 10$, and dots show the steady-state solution derived by Ogilat *et al.* [75]. The plot was generated for $P = 0.01$, $F = 0.5$ and with 60 complex roots of function $G_+(k, s)$ 132
- 5.8 Free surface profile in dimensionless variables relative to unperturbed level $y = 1$ for $t = 1000$ as obtained from equation (5.71) with fixed number of poles $N = 10$ from Table 5.1, but with different numbers of complex roots of function $G_+(k, s)$. Dashed line pertains to 40 roots, solid line – to 60 roots, and dots show the steady-state solution derived by Ogilat *et al.* [75]. The plot was generated for $P = 0.01$, $F = 0.5$ 133
- 5.9 The free surface profile for $P = 0.01$, $F = 0.5$ and different instants of time as per equation (5.71). 134

5.10	The free surface profile for $P = 0.01$, $F = 0.5$ and different instants of time as per equation (5.71). The steady state solution derived in Ogilat <i>et al.</i> [75] is shown in this figure for the comparison.	135
5.11	Maximal wave amplitude $A = \eta_{max}$ against time as per equation (5.71). The plot was generated for $P = 0.01$, $F = 0.5$	136
6.1	A sketch of the two-layer model with internal waves at the interface.	138
6.2	This figure shows the interface profile for different values of a : black line – $a = 0$, blue line – 0.5, red line – 0.7. Panel (a) – Froude number $F = 0.5$, and panel (b) – $F = 0.7$	149
6.3	This figure shows the dependence of wave amplitude (a) and wavelength (b) on the density ratio for two values of Froude number: $F = 0.5$ (red line) and $F = 0.7$ (black line).	150
C.1	Zero isolines of functions $\text{Ref}_1(k, z/t)$ (blue lines) and $\text{Im}f_1(k, z/t)$ (red lines). Dots indicate intersection points of isolines. The plot was generated for $F = 0.5$, $t = 1$ and $z_2 = 4.0277 - 1.1939i$	161
C.2	Zero isolines of functions $\text{Ref}_1(k, z/t)$ (blue lines) and $\text{Im}f_1(k, z/t)$ (red lines). Dots indicate intersection points of isolines. The plot was generated for $F = 0.5$, $t = 1$ and $z_3 = 3.2838 + 3.5944i$, $z_4 = 3.2838 - 3.5944i$ in figure (a) and (b) respectively.	162
C.3	Zero isolines of functions $\text{Ref}_1(k, z/t)$ (blue lines) and $\text{Im}f_1(k, z/t)$ (red lines). Dots indicate intersection points of isolines. The plot was generated for $F = 0.5$, $t = 1$ and $z_5 = 1.7154 + 6.0389i$, $z_6 = 1.7154 - 6.0389i$ in figure (a) and (b) respectively.	162
C.4	Zero isolines of functions $\text{Ref}_1(k, z/t)$ (blue lines) and $\text{Im}f_1(k, z/t)$ (red lines). Dots indicate intersection points of isolines. The plot was generated for $F = 0.5$, $t = 1$ and $z_7 = -0.8944 + 8.5828i$, $z_8 = -0.8944 - 8.5828i$ in figure (a) and (b) respectively.	163
C.5	Zero isolines of functions $\text{Ref}_1(k, z/t)$ (blue lines) and $\text{Im}f_1(k, z/t)$ (red lines). Dots indicate intersection points of isolines. The plot was generated for $F = 0.5$, $t = 1$ and $z_9 = -5.1612 + 11.3752i$, $z_{10} = -5.1612 - 11.3752i$ in figure (a) and (b) respectively.	163

List of Tables

3.1	The coordinates of key points in the f -plane.	58
3.2	The mapping between the z -, f - and ζ -planes	59
5.1	The first ten values of poles z_j and residues C_j provided by Trefethen <i>et al.</i> [57] through the Matlab program.	121
B.1	This table shows calculation of the exact values of the ratio $\Gamma(\mu_m)/\Gamma(\frac{1}{2} + \mu_m)$, and the approximation values by using Stirling's formula, for differ- ent values of the integer m , when the Froude number $F = 0.5$	156
B.2	This table shows the calculation of the real and imaginary part of the infinite product $T(\mu_R)$, for different values of the Froude number F , for the first N terms.	157
B.3	This table shows the calculation of the real and imaginary part of the infinite product $T(\mu_R)$, for different values of the Froude number F after N terms, given by equation (2.80).	158
B.4	This table shows the calculation of the infinite product $T(i\pi\mu_m)$, for different values of the Froude number F , given by (2.81). The calculation been made after N terms for different values of m	159